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The curve of double curvature, or *curve of tresines,* defined by the equations

$$x = \text{cotr}\,[\phi, o]$$
$$y = \text{tres}\,[\phi, o]$$
$$z = \text{tres}\,[o, \phi]$$

appears likewise to be most fertile in properties analogous to those of the circle. The following were stated by Mr. Graves:

1. *The angle between the tangents at any two points on the curve is equal to that between the two corresponding radii.*

2. *The angle between the osculating planes at any two points on the curve is equal to the angle between the radii drawn to the two reciprocal points.*

3. *The angle of contact at a point on the curve is therefore equal to the torsion at the reciprocal point.*

4. *The element of the curve described by the reciprocal point is double the elementary area described by the radius vector.*

5. *The polar reciprocal of the developable surface formed by the osculating planes is itself* the curve of tresines.

6. *The product of the radii of curvature at any point and at its reciprocal is equal to unity.*

7. *The radius vector traces a logarithmic spiral upon a plane parallel to the symmetric plane.*

Mr. John Neville read a paper on the maximum Amount of Resistance required to sustain Banks of Earth and other Materials.

Let CDE be any bank of earth, sand, or other material, and CE the position of the line of repose with respect to the horizon and bank; then if we put CA, the perpendicular from C on DE produced $= h$, the $\angle$ DEA $= b$; the angle ECA $= c$; and the $\angle$ ACF, which the fracture CF makes with CA, $= \phi$; then when the horizontal resistance R, necessary to support the wedge FCD, is a maximum,*

$$\tan\phi = \frac{\sec.c\sqrt{\tan b\tan c + 1} - 1}{\tan c} \qquad (1)$$

and

$$\text{R} = \frac{h^2\text{W}}{2}\left(\frac{\sec c - \sqrt{\tan b\tan c + 1}}{\tan c}\right)^2, \qquad (2)$$

where $\text{W} =$ the weight of a cubical unit of the stuff in FDC. Equation (2) may be changed into the following,

$$\text{R} = \frac{h^2\text{W}}{2\tan c}\left(\sqrt{\tan c + \cot c} - \sqrt{\tan b + \cot c}\right)^2 \qquad (3)$$

a simple form to calculate the values of R from.

Also if CO is drawn parallel to DE, and we put

$$\theta = \angle\ \text{GDE} = \angle\ \text{DCO};\ \ \delta = \angle\ \text{OCE} = \theta - (c - b),$$

and $\text{H} =$ CD the slant height of the face: then

$$\tan(c - \phi) = \tan\angle\ \text{FCE} = \sqrt{\tan^2\delta + \tan d\tan\delta} - \tan\delta \qquad (4)$$

and

$$\text{R} = \frac{\text{H}^2\text{W}}{2} \times \frac{\tan\theta}{\tan\theta - \tan d} \times \tan^2(c - \phi), \qquad (5)$$

where $d = c - b = \angle$ ECD.

When $b = 0$, CD coincides with CA, and we get from equation (1)

$$\tan\phi = \frac{\sec c - 1}{\tan c} = \tan\tfrac{1}{2}c,$$

which shews that the equation of Prony holds good, whatever the inclinations of CD and DE to the horizon may be, if these lines continue at right angles to each other. We also get from equation (2) or (5) in this case,

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* Generally we have $\text{R} = \text{W} \times \text{DCF} \times \tan.\ \text{FCE}$, for any fraction CF; a very simple expression for finding the value of the resistance when the position of the fracture is given.

$$\text{R} = \frac{\text{WH}^2}{2} \tan^2 \tfrac{1}{2} c, \qquad (6)$$

which shews the value of R retains the same form whatever the inclination of CD may be, if DE continues at right angles to it, as may also be seen, in the more general case, from equation (2) or (5).

When $\theta = c - b$, DE stands at the angle of repose, and when infinite, from equation (4), the fracture CF becomes parallel to it; and from equation (2) or (5) may be had, by reduction,

$$\text{R} = \frac{\text{WH}^2}{2} \times \sin^2 (c-b). \qquad (7)$$

As in loose stuff DE can never stand at a steeper inclination than the angle of repose, equation (7) gives the greatest value R can ever attain; the height of the face H being constant. When $b = 0$, H vertical, and $c = 34°$, which corresponds to a slope of repose of $1\frac{1}{2}$ to 1 nearly, we get, by comparing equations (6) and (7),

R in (6) : R in (7) : : 283 : 687 : : 3 : 7, nearly.

In fluids the ratio will be as 1 to 1, and when the angle of repose is 90°, the ratio will be as 1 to 4; for $\tan^2 \frac{1}{2} c : \sin^2 c : : \frac{1}{2} c^2 : c^2 : : 1 : 4$: hence no instance can occur wherein, the face H being vertical, the value of R can exceed four times the value when the top DE is horizontal.

Equation (4) gives the following simple geometrical construction for finding the line CF. Draw any line MO, cutting the line of repose CE at right angles in K, and terminating in the face CD at M, and in the line CO parallel to DE, the top, in O. On MO describe a semicircle, cutting CE, the line of repose, in H; from O as centre, with OH as radius, describe an arc to cut OM in I; join C to I, and produce CI to F; then the wedge FDC will require the maximum resistance. When DE is at right angles to DC, the angle ECD is bisected by EF.

The equations apply also to finding the value of R and position of CF for "dwarf walls" CD, at the toes of cuttings or embankments; but when the fracture CF lies inside the top of the slope DG, the value of R and position of CF are found as follows:

Produce EG to A, and draw CA so that $\Delta COD = \Delta GOA$, and as, therefore, $\Delta FAC = FGDC$, we have only to determine R, and draw CF as if the bank to be supported was CAE.

In the Encyclopædia Britannica, seventh edition, article "*Masonry*," by Tredgold, there are given the following equations:

$$\text{“(23)} \quad \tan a = \frac{-1 + (\tan c \tan^3 i + \tan^2 i + \tan c \tan i +)^{\frac{1}{2}}}{\tan i}\text{”}$$

$$\text{“(25)}\ \mathrm{R} = \frac{h^2 s}{2\tan i}\left[\tan i + \tan c + \frac{2}{\tan i} - 2\left(\tan c \tan i + \frac{\tan c + 1}{\tan i} + 1\right)^{\frac{1}{2}}\right]\text{”}$$

where $a = \angle ECa$, which the plane of fracture CE makes with the vertical line Ca; $c = \angle ACa$; $i =$ the complement of the angle of repose; $h =$ AC the vertical height, and S $=$ the weight of a cubical foot of the material in ACE, AE being ***horizontal:*** and the following table of calculated values of R from equations (26) and (25):

		Angle of Repose.	Weight of S.	Value of R. when $c = 0°$	Value of R when $c = 10°$
1	Water, ...	0°	62·5lbs.	$R = 31\frac{1}{4}h^2$	$R = 31\frac{1}{4}h^2$
2	Fine dry sand, .	33°	92 —	$R = 13·8h^2$	$R = 4·8h^2$
3	Do. moist .	——	119 —	$R = 17·85h^2$	$R = 6·2h^2$
4	Quartz sand, dry,	35°	102 —	$R = 13·77h^2$	$R = 4·6h^2$

Now, using the same data, and calculating from equations (6) and (2) here given, the values will be as follow:

		Angle of Repose.	Weight of W.	Value of R. when $b = 0$, and C D vertical: equation (6).	Value of R when $b = 10°$ and D E horizontal: equation (2).
1	Water, ...	0°	62 5lbs.	$R = 31\frac{1}{4}h^2$	$R = 31\frac{1}{4}h^2$
2	Fine sand, dry, .	33°	92 —	$R = 13·5h^2$	$R = 9·7h^2$
3	Do. moist, .	——	119 —	$R = 17·5h^2$	$R = 12·6h^2$
4	Quartz sand, dry,	35°	102 —	$R = 13·8h^2$	$R = 9·7h^2$

The values in the last column have been calculated from equations (2) and (5) respectively, which gave the same results when the latter is multiplied by $\sec^2 b$, as $h^2 \times \sec^2 b = H^2$: they are fully double those given by Tredgold, and made use of by him in calculating the dimensions of retaining walls given in another Table. The differences in the Tables, when $b = 0$, are immaterial. Equation (25) is, however, incorrect, as will be seen by squaring the second part of equation (3) here given, actually, when we get

$$R = \frac{h^2 w}{2\tan c}\left(\tan c + \tan b + \frac{2}{\tan c} - 2\sqrt{\tan c \tan b + 1 + \frac{\tan c \tan b + 1}{\tan^2 c}}\right)$$

Whence it appears that the term $\frac{\tan c + 1}{\tan i}$, in Tredgold's equation (25) under the square root, should be $\frac{\tan c \tan i + 1}{\tan^2 i}$; this being the case, his equation is then reducible to equation (3). Equation (23) can also be reduced to the form of equation (1), which is general, whether DE is horizontal or inclined.